

NEW MECHANISM OF CP -VIOLATION IN THE MODELS OF ELECTROWEAK BARYOGENESIS

Andro BARNAVELI and Merab GOGBERASHVILI

Institute of Physics, Georgian Acad. Sci.

Tamarashvili str. 6, Tbilisi 380077, Republic of Georgia.

(E-mail: bart@physics.iberiapac.ge ; gogber@physics.iberiapac.ge).

Abstract

The new source of CP -violation in the frames of "charge transport" mechanism of electroweak baryogenesis is investigated. The consideration is based on the assumption that C and CP need not be violated at the same place, and on the fact that in the case of nonpolarized flux of fermions falling nonperpendicularly on the phase-separating domain wall the transmitted flux will be polarized (i.e. P and CP are violated). Sphaleron processes can convert the (P and CP)-asymmetric state into (C and CP)-asymmetric one. We argue that the value of CP -violation in such mechanism can explain the observed value of Baryon Asymmetry of the Universe even in the frames of the Minimal Standard Model.

1 Introduction

Almost all models describing generation of Baryon Asymmetry of the Universe (BAU) on the electroweak scale use different extensions of the Minimal Standard Model (MSM), since the value of C - and CP -violation (which is necessary for baryogenesis) following from Kobayashi-Maskawa matrix is too small to explain the observed value of BAU [1, 2]

$$\delta = \frac{n_B}{s} \sim 10^{-11} \div 10^{-8} \quad (1.1)$$

(where n_B is the net baryon density and s is the entropy density). The only exception is the model elaborated in works [3, 4], which tries to explain BAU in the frames of MSM and uses temperature effects to yield the sufficient CP -violation. However it causes a lot of discussions [5] and the question remains open.

In this paper we consider the other possible source of C - and CP -violation which arises during the first order electroweak phase transitions even in the frames of MSM.

We are basing on the following moments:

1. *For the BAU generation C - and CP -violation need not occur at the same place.* This assumption is analogous to that given in the so-called "charge transport" mechanism [6, 7], where it was considered that B - and CP -violation take place during the first order phase transitions in the different areas: CP -violation — exactly on the phase-separating domain wall, while B -violation — in the symmetric phase.
2. *Interaction of particles with domain wall can violate P - and CP -symmetry.* Indeed, as we will see, if one considers the nonpolarized flux of fermions (i.e. the equal numbers of positive- and negative-helicity fermions) falling not perpendicularly upon a phase-separating domain wall, then the transmitted flux will be polarized i.e. (P and CP)-asymmetric. Note, that the P -violating features of domain walls are well known (see [8] and references therein). Here we would like to mention also, that the phenomenon of fermion flux polarization by the phase boundary is in some sense analogous to the "Mallus effect" in optics [9] when the nondirect sun rays are polarized by the atmosphere.
3. *Any P -asymmetry inside the new-phase bubble is eliminated due to multiple interactions of massive particles and with account of spherical symmetry of the problem.*
4. *The anomalous B -violating sphaleron processes in the symmetric phase can convert the axial asymmetry into baryon asymmetry [7].* Thus, effectively, (P and CP)-asymmetry (created on the domain wall) is converted (in the symmetric phase) into (C and CP)-asymmetry.

Here we would like to note that some misunderstanding can take place, since the bubble solutions are spherically symmetric and thus, globally, no P -violation must occur. However nucleation of bubbles creates the distinguished frame, for example, the rest frame of the bubble wall and violates Lorentz-invariance. Namely in this frame parity is violated and such "local" P -violation plays the crucial role in our mechanism. Besides, due to processes inside and outside the bubbles this P -asymmetry is eliminated or converted into (C and CP)-asymmetry.

Thus, globally, no P -asymmetry remains and this problem must not arise.

In the next section we describe shortly the mechanism of baryogenesis.

In section 3 the parity violation in the fermion – wall interactions is considered.

In section 4 the numerical estimation of BAU, generated through the described mechanism, is given.

2 The mechanism of baryogenesis.

The mechanism of baryogenesis, described above, works at the first order phase transitions. Such phase transitions proceed via nucleation and expansion of the new (broken) phase bubbles within the old (symmetric) one [1, 11, 10]. These bubbles grow until the universe is completely converted to the broken phase.

During their motion through the plasma the bubble walls interact with the particles. This interactions violate P and CP . Namely, for the fermions falling nonperpendicularly upon the wall the latter changes the polarization of initial flux. With the natural assumption that the equal number of right- and left-handed fermions from the old phase are colliding with the phase boundary this will cause the polarized flux of particles penetrating into the broken phase (i.e. more fermions of, say, negative helicity enter the new phase through the bubble wall).

Since the mean radius of bubbles (to the end of the phase transition) $R \sim 10^{12}/T$ [10] is by many orders greater than the particle mean free path $l \sim (4 \div 10)/T$ [1], the fermions penetrated into the new phase bubble can reach its walls again only after collisions with other particles. This interactions of massive particles (with account of spherical symmetry of the problem) eliminate the helicity-asymmetry (i.e. P -asymmetry) inside the bubbles and one can assume that equal numbers of positive- and negative-helicity fermions (i.e. nonpolarized flux) from the new phase do reach the bubble walls.

Now let us see, what does happen with the nonpolarized flux of fermions falling upon the phase boundary from the new phase, i.e. from inside the bubble (of course, due to the wall motion, this flux is smaller than that from the unbroken phase). Due to polarization-changing features of domain walls the polarized chiral (or "axial") flux

of fermions coming out of the bubble into the symmetric phase will occur. The amount of right-handed fermions (which are isosinglets) and anti-fermions (which are isodoublets) entering the symmetric phase will be greater than the amount of left-handed fermions (which are isodoublets) and anti-fermions (which are isosinglets) entering it. Note that for the massless particles in the symmetric phase the definitions of "helicity" and "chirality" are equivalent; the positive helicity corresponds to the right chirality and negative helicity — to the left chirality. Thus one can say that the new phase bubble emits into an unbroken phase a (P and CP)-asymmetric distribution of particles.

The particle distribution functions in front of the wall approach local thermal and chemical equilibrium, characterized by a temperature and by chemical potentials for the various conserved and approximately conserved charges. Because isosinglet and isodoublet particles carry different values of weak hypercharge, which is a conserved quantum number in the unbroken phase, the nonzero "axial" flux from the wall will lead to a nonzero density of fermionic hypercharge Y in the region preceding the wall [7]. Using of this quantum number is more convenient, since the "axial charge" is not a good quantum number, as it is violated by Higgs scattering [1]. Note, that with account of charge-screening effects it will be more correct to consider not the fermionic hypercharge Y , but rather the linear combination $B' = B - xY$, where B is the baryon number and x is chosen so that B' is not screened by gauge forces (calculations show that $x \sim 1/5$ [1, 7]). However this does not change the results significantly.

As it was shown [1, 6, 7, 12], in the presence of nonzero hypercharge the free energy of the unbroken phase is minimized for nonzero baryon number. Anomalous baryon violating electroweak processes will be biased favoring the production of baryon number over anti-baryon number. Therefore the (P and CP)-asymmetric state will be converted into (C and CP)-asymmetric state.

Then this baryon asymmetry, created in the symmetric phase, will eventually pass into the expanding bubbles of the broken phase, where baryons are stable, and is conserved up to day.

3 Interaction of fermions with the phase boundary.

We have noted above, that interactions of fermions with the phase boundary violate (P and CP)-invariance. This is exhibited in the difference between polarizations of the initial and the transmitted fluxes. Let us discuss this subject in more details.

Fermions are interacting with the scalar field, forming the bubble, through the Yukawa coupling. Since in the old phase the vacuum expectation value (VEV) of the scalar field $\langle \varphi \rangle = 0$ and in the new phase $\langle \varphi \rangle \neq 0$, the mass of fermions is spatially varying. In fact, one has to deal with the Dirac equation with spatially varying mass term.

$$(\gamma_\mu \partial^\mu - m(z)) \psi = 0. \quad (3.1)$$

To solve such equation we shall make some simplifications.

First of all, since the bubble radius is by many orders larger than the particle mean free path, we can consider the bubble wall as the planar one (e.g. in the x - y plane) and to assume, that it moves in the negative z direction. The second, at the particular choice of parameters, in MSM, the wall width can be $\delta_w \sim O(1/T)$ [11], i.e. smaller, than the particle mean free path. Then one can neglect δ_w and consider the wall as infinitely thin. In this case one has:

$$m(z) = \begin{cases} 0, & z < 0 \\ m, & z > 0. \end{cases} \quad (3.2)$$

In the rest frame of the bubble wall the problem is equivalent to the quantum mechanical problem of fermion scattering from the rectangular potential barrier [13]. We shall investigate the nonperpendicular scattering. Note, that in all models of "charge transport" mechanism [6, 7] and also in [4] only normal scattering of particles from the phase boundary was considered, while namely nonperpendicular scattering effects give the nonzero contribution to the CP -violation.

Let us introduce the new parameter

$$\eta \equiv \frac{E - m}{k} = \frac{k}{E + m} = \sqrt{\frac{E - m}{E + m}} \quad (3.3)$$

(where E is the fermion energy and \vec{k} — its momentum), the angle θ characterizing the direction of \vec{k} :

$$k_t \equiv k_x = k \sin \theta \quad ; \quad k_l \equiv k_z = k \cos \theta \quad (3.4)$$

(where k_l, k_t are the momenta perpendicular and parallel to the wall respectively; due to cylindrical symmetry of the problem the direction of \vec{k} does not depend on the polar angle ϕ and one can consider scattering in x-z plane), and denote the incident, reflected and transmitted waves as ψ , ψ' and ψ'' respectively, while their wave vectors as \vec{k} , \vec{k}' and \vec{k}'' and the angles characterizing their direction as θ , θ' and θ'' respectively. At the phase boundary ($z = 0$) the following condition must be satisfied

$$\psi + \psi' = \psi''. \quad (3.5)$$

This implies that

$$\begin{aligned} \theta' &= \pi - \theta \\ k \sin \theta &= k'' \sin \theta''. \end{aligned} \quad (3.6)$$

Let us assume that the incoming flux of particles is nonpolarized i.e. equal amounts of positive- and negative-helicity fermions are falling upon the wall. Then in the standard basis

$$\psi = \frac{1}{\sqrt{V(1+\eta^2)}} \cdot \begin{pmatrix} \left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right) \\ \left(\sin \frac{\theta}{2} + \cos \frac{\theta}{2} \right) \\ \eta \left(\cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right) \\ \eta \left(\sin \frac{\theta}{2} - \cos \frac{\theta}{2} \right) \end{pmatrix} \cdot e^{-i(Et - \vec{k}\vec{r})}. \quad (3.7)$$

Here V is the volume of normalization and we use the signature $(+, -, -, -)$. The reflected and the transmitted waves can be written in the form

$$\psi' = \frac{1}{\sqrt{V(1+\eta^2)}} \cdot \begin{pmatrix} \left(-C \sin \frac{\theta}{2} + D \cos \frac{\theta}{2} \right) \\ \left(C \cos \frac{\theta}{2} + D \sin \frac{\theta}{2} \right) \\ \eta \left(C \sin \frac{\theta}{2} + D \cos \frac{\theta}{2} \right) \\ \eta \left(-C \cos \frac{\theta}{2} + D \sin \frac{\theta}{2} \right) \end{pmatrix} \cdot e^{-i(Et - \vec{k}'\vec{r})}; \quad (3.8)$$

$$\psi'' = \frac{1}{\sqrt{V(1+\eta^2)}} \cdot \begin{pmatrix} \left(G \cos \frac{\theta''}{2} - F \sin \frac{\theta''}{2} \right) \\ \left(G \sin \frac{\theta''}{2} + F \cos \frac{\theta''}{2} \right) \\ \eta'' \left(G \cos \frac{\theta''}{2} + F \sin \frac{\theta''}{2} \right) \\ \eta'' \left(G \sin \frac{\theta''}{2} - F \cos \frac{\theta''}{2} \right) \end{pmatrix} \cdot e^{-i(Et - \vec{k}''\vec{r})}, \quad (3.9)$$

respectively, where the coefficients C and G characterize the states with positive helicity, while coefficients D and F — states with negative helicity and $\eta = \eta'$. Substituting equations (3.7),(3.8),(3.9) into boundary condition (3.5) one can yield [13]:

$$C = \frac{-\alpha + \beta}{\Delta} \quad ; \quad D = -\frac{\alpha + \beta}{\Delta} \quad ; \quad (3.10)$$

$$G = \frac{\rho + \sigma}{2\xi\lambda} \quad ; \quad F = \frac{\rho - \sigma}{2\xi\lambda} \quad ; \quad (3.11)$$

where

$$\begin{aligned} \alpha &= 2(\lambda^2 + 1)p(1 + q^2) - 4\lambda q(1 + p^2), \\ \beta &= (\lambda^2 - 1)(1 - p^2)(1 + q^2), \\ \rho &= \frac{1}{\Delta} \cdot 4\lambda(\lambda + 1)(1 - pq)(1 - p^2), \\ \sigma &= \frac{1}{\Delta} \cdot 4\lambda(\lambda - 1)(p + q)(1 - p^2), \\ \Delta &= (\lambda + 1)^2(1 - pq)^2 + (\lambda - 1)^2(p + q)^2, \\ p &\equiv \tan \frac{\theta}{2} \quad , \quad q \equiv \tan \frac{\theta''}{2} \quad , \\ \xi &= \sqrt{\frac{1 + \eta^2}{1 + \eta'^2}} \cdot \frac{\cos \frac{\theta''}{2}}{\cos \frac{\theta}{2}}; \\ \lambda &= \frac{\eta''}{\eta}. \end{aligned} \quad (3.12)$$

It is easy to understand that polarization of reflected and transmitted fluxes are

$$\begin{aligned} h' &= \frac{C^2 - D^2}{C^2 + D^2} = -\frac{2\alpha\beta}{\alpha^2 + \beta^2}, \\ h'' &= \frac{G^2 - F^2}{G^2 + F^2} = \frac{2\rho\sigma}{\rho^2 + \sigma^2}. \end{aligned} \quad (3.13)$$

If one considers the flux of particles coming towards the phase boundary from the unbroken phase, then $\eta = \eta' = 1$, $\eta'' < 1$. In this case, with account that $0 < \theta/2 < \pi/4$ i.e. $p < 1$ and $q < 1$, one yields: $\alpha = 0$, $\lambda < 1$, $\rho > 0$, $\sigma < 0$. This means that $h' = 0$ and $h'' < 0$, i.e.

the reflected flux will be nonpolarized, while the transmitted flux will have a negative polarization.

Now, if one considers the flux of particles falling on the wall from the new phase, then $k = k' = \sqrt{E^2 - m^2}$, $k'' = E$, $\eta = \eta' < 1$, $\eta'' = 1$. Then $\lambda > 1$, $\sigma > 1$, and $h'' > 0$. This means that the positively polarized flux of fermions (i.e. more right-handed fermions than those left-handed) will be emitted from the new phase bubble into the old phase.

Note that if the initial flat wave falls on the wall perpendicularly, then $\theta = \theta'' = 0 \Rightarrow p = q = 0 \Rightarrow \alpha = \sigma = 0 \Rightarrow h' = h'' = 0$ and the polarization of reflected and transmitted fluxes will be equal to that of the initial flux.

Now let us calculate the coefficients of reflection and transmission through the phase boundary. We can express the current density

$$j_\mu = ie\bar{\psi}\gamma_\mu\psi \quad (3.14)$$

in terms of spinor components. Using (3.7) – (3.12) one can obtain that

$$\begin{aligned} j_x &= \frac{4e\eta}{V(1+\eta^2)} \cdot \sin\theta; \\ j_z &= \frac{4e\eta}{V(1+\eta^2)} \cdot \cos\theta. \end{aligned} \quad (3.15)$$

$$\begin{aligned} j'_x &= \frac{2e\eta}{V(1+\eta^2)} \cdot (C^2 + D^2) \sin\theta; \\ j'_z &= -\frac{2e\eta}{V(1+\eta^2)} \cdot (C^2 + D^2) \cos\theta. \end{aligned} \quad (3.16)$$

$$\begin{aligned} j''_x &= \frac{2e\eta''}{V(1+\eta''^2)} \cdot (G^2 + F^2) \sin\theta''; \\ j''_z &= \frac{2e\eta''}{V(1+\eta''^2)} \cdot (G^2 + F^2) \cos\theta''. \end{aligned} \quad (3.17)$$

Let us define the "full coefficient of reflection" as the ratio of reflected current density to the incident one:

$$R \equiv \frac{\sqrt{j'^2_x + j'^2_z}}{\sqrt{j^2_x + j^2_z}} = \frac{C^2 + D^2}{2}, \quad (3.18)$$

and the "full coefficient of transmission" as the ratio of transmitted current density to the incident one:

$$K \equiv \frac{\sqrt{j_x''^2 + j_z''^2}}{\sqrt{j_x^2 + j_z^2}} = \frac{\eta''(1 + \eta^2)}{\eta(1 + \eta'^2)} \cdot \frac{(G^2 + F^2)}{2}. \quad (3.19)$$

4 Computation of BAU.

Now we can compute the net flux of axial number coming from the new phase bubble into the symmetric phase.

Taking into account, that both — the left-handed fermions which are isodoublets and the right-handed fermions which are isosinglets, and also their antiparticles – left-handed antifermions which are isosinglets and right-handed antifermions which are isodoublets, can be emitted into unbroken phase (we denote the plasma-frame fluxes of these species as (f_L^d) , (f_R^s) , (f_L^s) , (f_R^d) , respectively), the flux density of axial number into the symmetric phase in the plasma frame will be described by the expression (compare with [7, 12]):

$$\begin{aligned} f_A &= f_L^d + f_L^s - f_R^s - f_R^d = \\ &= \frac{2}{\gamma} \int_0^\infty dk_l \int_0^\infty \frac{k_t dk_t}{4\pi^2} \frac{k_l}{k} \cdot \{e^{\gamma \cdot (E + v_w k_l)/T} + 1\}^{-1} \cdot [K_+ - K_-], \end{aligned} \quad (4.1)$$

where v_w is the wall velocity, γ is the Lorentz-factor and

$$K_+ - K_- = h'' K = \frac{\eta''(1 + \eta^2)}{\eta(1 + \eta'^2)} \cdot \frac{(G^2 - F^2)}{2}. \quad (4.2)$$

(see also (3.3), (3.11), (3.12)), while all quantities in these formulae can be expressed via k_l and k_t . Since the isosinglets and isodoublets possess the different values of weak hypercharge ($Y_d = 4/3$, $Y_s = 1/3$), which is conserved in symmetric phase, the flux density of weak fermionic hypercharge emitted from the bubble into the old phase will be

$$f_Y = \frac{4}{3} (f_L^d - f_R^d) + \frac{1}{3} (f_R^s - f_L^s) = \frac{1}{2} f_A. \quad (4.3)$$

As it was shown in [7, 12], a nonzero hypercharge biases an anomalous weak interactions towards production of baryons rather than antibaryons. This process converts the (P and CP)-asymmetric state

into (C and CP)-asymmetric one. The net baryon number produced in a unit volume

$$n_B \sim \frac{6\Gamma_B}{T^3} \cdot \frac{f_Y \tau_T}{v_w}, \quad (4.4)$$

where

$$\Gamma_B \sim a(\alpha_W T)^4$$

is the rate per unit volume of baryon violating transitions, factor $a \sim (0.1 \div 1.0)$, $\alpha_w^4 \sim 10^{-6}$, τ_T — "transport time", i.e. the average time each transmitted fermion spends in the old phase before being struck by the bubble wall for a second time. Using the expression for entropy

$$s = 2\pi^2 g_* \frac{T^3}{45},$$

where $g_* \sim 100$ is a statistical factor, for the baryon-to-entropy ratio produced during the phase transition finally we obtain:

$$\delta = \frac{n_B}{s} \sim \frac{6a\alpha_W^4 T f_Y \tau_T}{s v_w} \sim \frac{150a\alpha_W^4}{\pi^2 g_*} \cdot \frac{f_Y \tau_T}{v_w T^2}. \quad (4.5)$$

Let us evaluate this quantity. If we consider a *top*-quark as a fermion, responsible for BAU generation, then numerical estimations show, that $\tau_T \sim O(100/T)$ [1]. The bubble walls with a width smaller than a particle mean free path move with a velocity $v_w \sim 0.04$ [11]. The phase transition takes place at temperature $T \sim 10^2 GeV$. Thus we can write that

$$\delta \sim (10^{10} \div 10^{-11})(GeV)^{-3} \cdot f_A. \quad (4.6)$$

Numerical estimations of the integral (4.1) give

$$f_A \sim (10^3 \div 10^4)(GeV)^3, \quad (4.7)$$

As it is easy to see, this value does correspond to the observed value (1.1).

Thus we can conclude, that the rate of CP -violation through the mechanism described above is sufficient enough to explain the BAU. Since all calculations were produced in the frames of MSM, it means that there is no reason to take different extensions of this model in

search of extra source for CP -violation. However still remains the problem of too small Higgs mass ($m_H < 45 GeV$) necessary for the produced BAU not to be eliminated by the B -violating processes in the broken phase. Thus the question of validity of MSM for baryogenesis remains open.

Modifications of the CP -violation mechanism described above can be applied also to the models of "spontaneous baryogenesis" [1], of baryogenesis from cosmic strings [14] and other models. We are going to investigate them in future papers.

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